ME424 Fall 2021 **Homework # 2**

Assigned September 16, 2020 Due: Friday, September 24, 2020

- Submit your solution to BB before the deadline
- To receive credits, please write down all the necessary steps leading to final answer.

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1. *Column and Null Space*: Define

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ -1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} -2 & -1 & 1 \\ 1 & 5 & 4 \\ 1 & -1 & -2 \\ 1 & 2 & 1 \\ 1 & 5 & 4 \end{bmatrix}$$

   (a) What are the dimensions of the null space and column space (i.e. range space) of A?

   (b) Find a set of basis vectors for null(A).

   (c) Find a set of basis vectors for col(A)

   (d) Is col(C) = col(A)? Justify your answer.

   (e) Find a matrix B of appropriate dimension such that $C = AB$. (You should be able to find B just by inspection).

   Hint: Let a_1, a_2 be the columns of A and $c_1, \ldots, c_3$ be the four columns of C. By inspection (simple calculation), the following relations hold

$$c_1 = -a_1 + a_2, \quad c_2 = a_1 + 2a_2, \quad c_3 = 2a_1 + a_2$$

2. *Speak the Matrix Language*: Express the following statements in matrix language. You can assume that all matrices mentioned have appropriate dimensions. Here is an example: "Every column of C is a linear combination of the columns of B" can be expressed as"$C = BF$ for some matrix F". There can be several answers; one is good enough. You are expect to justify all of your answers.

   (a) For each i, row i of Z is a linear combination of rows $i, \ldots, n$ of Y.

   (b) W is obtained from V by permuting adjacent odd and even columns (i.e., 1 and 2, 3 and 4,...).

   (c) Each column of P makes an acute angle with each column of Q.

   (d) Each column of P makes an acute angle with the corresponding column of Q.

   (e) The first k columns of A are orthogonal to the remaining columns of A.

3. *Matrix Rank*:

   (a) Let $a \in \mathbf{R}^n$ be an n-dim vector. Show that the $n \times n$ matrix $A \triangleq aa^T$ is of rank 1.

(b) Given two nonzero square matrices $A \in \mathbf{R}^{n\times n}$ and $B \in \mathbf{R}^{n\times n}$, argue that if $AB = 0$, then neither A nor B can be full rank.

(c) Explain why the system $Ax = b$ has a solution if and only if $\text{rank}(A) = \text{rank}\left([A \;\; b]\right)$.

4. Find a set of basis vectors of the plane (subspace) that is perpendicular to the line $x+y-z = 3$ in R^3

5. Find the matrix that projects every point in the plane onto the line $x - 2y = 0$.