MEE5114 Spring 2022 **Homework # 2**
Assigned Feb 28, 2022 Due: March 9th, 2022

1. Show that for any matrix $A \in \mathbb{R}^{n\times n}$, the infinite series $e^A = I + A + \frac{A^2}{2!} + \cdots$ converges.

2. Given a linear system $\dot{x} = Ax + Bu$. Its solution is given by

$$x(t) = e^{At}x_0 + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

Now assume we have $u(t) \equiv u_k$ for $t \in [k\delta t, (k+1)\delta t)$. Please derive the Zero-order-hold discretization rule, namely, derive expressions for A_d and B_d such that

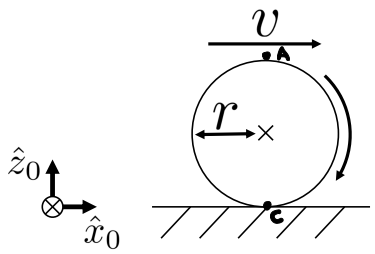
$$x_{k+1} = A_d x_k + B_d u_k$$

where $x_k \triangleq x(k \cdot \delta t)$ and $u_k = u(k \cdot \delta t)$

3. **Spatial Velocity:** A cylinder rolls without slipping in the $\hat{x}_0$ direction on the $\hat{x}_0 - \hat{y}_0$ plane. The cylinder has a radius of r and a constant forward speed of v. Let ${}^0C=[C_x(t),0,0]^T$ be the position of the contact point at time t. Let ${}^0A=[A_x(t),0,0]^T$ be the position of the instantaneous top of the cylinder at time t.

   (a) What is the linear velocity of the point C? (hint: just need to compute $\frac{d}{dt}C_x(t)$)?
   (b) What is the linear velocity of the point A?
   (c) What is velocity of the body-fixed point currently coincides with C?
   (d) What is velocity of the body-fixed point currently coincides with A?
   (e) What is the spatial velocity of the cylinder in $\{0\}$-frame?
   (f) What is the spatial velocity of the cylinder in frame $\{C\}$? ($\{C\}$ has the same orientation as $\{0\}$, while its origin is at the contact point C)

   Note: The first 4 questions are all referring to the inertia frame $\{0\}$



4. **Spatial Velocity:** Modern Robotics: Exercise 5.5

5. **Screw axis and its transformation:**

   (a) Draw the screw axis for the twist $\mathcal{V} = (0, 2, 2, 4, 0, 0)$
   (b) Consider an arbitrary screw axis $\mathcal{S}$. Suppose the axis has gone through a rigid body transformation $T = (R, p)$ and the resulting new screw axis is $\mathcal{S}'$. Show that

$$\mathcal{S}' = [\mathrm{Ad}_T]\,\mathcal{S}$$

   (we have given the proof in class, you need to go through it on your own again)

(c) Consider a rigid body motion: rotation about z axis counterclockwise by 90^o and then translate along negative y-axis by 1m. All the axes are with respect to the fixed inertia frame.

i. Find the numerical values of the corresponding transformation matrix T;

ii. Move the screw axis in part (a) using T. Find the new screw axis $\mathcal{S}'$ after the motion.