ME424 Fall 2022 **Homework # 4**

Assigned October 23, 2022 Due October 28, 2022

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1. Check whether the discrete-time system with the following A matrix is asymptotically stable or not?

   (a) $A = \begin{bmatrix} 0.9 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) For the A matrix in part (a), please construct nonzero B and C matrices so that the overall system is BIBO.

2. **Controllability**: Considering the following system

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{1}$$

   (a) Is the system controllable?

   (b) Can you find a control input to steer the system from $x_0 = [0, 0, 0, 0]^T$ to $x_f = [1, 1, 1, 0]^T$? If not, explain why, otherwise find the control input sequence that drives the system from the origin to $\hat{x}$ within the minimum number of steps.

3. **Observability**: Considering the following system

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \tag{2}$$

   (a) Is the system observable?

   (b) Suppose $u(k) = 0$, for $k = 0, 1$ and $y(0) = [1, 2]^T, y(1) = [3, 4]^T$. Can you find two different initial states $x_0^{(1)}$ and $x_0^{(2)}$ that both agree with the given input output data.

4. Consider the motor speed control system under a discrete-time Proportional Integral (PI) controller as shown in Fig 1.

   (a) Find the state-space model for the PI controller (from the error to the Armature voltage)

   (b) Obtain the state-space model for the DC motor

   (c) Write a Python code to simulate the closed-loop system response under the unit step input. Try to find a set of K_p and K_I so that the response has zero steady-state error. Attach your codes and the successful simulation plots.

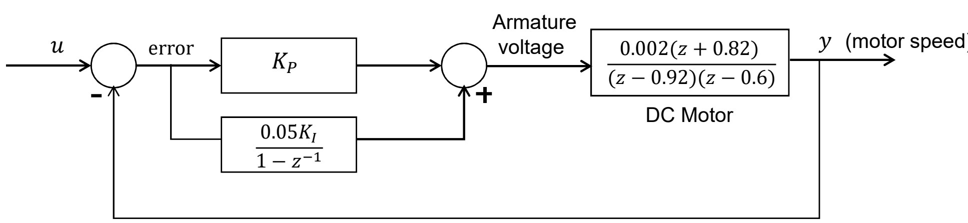

Figure 1: PI Controller for a DC Motor