ME424 Fall 2022 **Project # 1**

Assigned October 10, 2022 Due: October 20, 2022

- Please attach all of your codes and relevant figures. To receive credits, please write down all the necessary steps leading to your final solution. Make sure your discussions about the results are clear, brief, and to the point.
- Please work on your project independently. This course has a zero-tolerance policy on plagiarism.
- You must type your report. Overall presentation and organization count.

---

1. **Wind power forecast** concerns the estimation of expected power production of a wind farm. The forecasts are useful for turbine active control, power system management, energy trading, power plant scheduling, etc. *Short-term* and *long-term* forecasts (from a couple of hours to several days) are challenging and require various kinds of information such as temperature, seasonality, and wind speed and direction at nearby wind farms. In this project, we focus on *very short-term* forecast on wind speed using the tools learned in this course.

   Now suppose you are asked to develop an online wind-speed prediction algorithm that predicts wind speed 10 minutes into the future. Suppose that you decide to use an AR model of the following form:

   $$y(k) = a_1 y(k-1) + a_2 y(k-2) + \cdots a_n y(k-n) + v(k), \tag{1}$$

   and you want to determine both the number of regression terms n and the corresponding parameter value: $\theta^{(n)} = [a_1, a_2, \ldots, a_n]^{\mathrm{T}}$.

   (a) *Data:* Download the "winddata.csv" file from Carmen. Tutorial 2 provides you a method about how to read a ".csv" file in Python. Note that the second column of "winddata.csv" is the wind speed data (i.e., the measurement data $y(1), y(2), \ldots$) from a wind farm in Taxes. The unit used in the data is 100m/s. (1pts)

   (b) *Identification*: Write a program to identify the AR parameter θ using ordinary least squares for $n = 1$ to $n = 20$. Please attach the derivation of your model. For this part, please only use the first 5000 wind speed measurement data you obtained from part (a). For each value of n, compute the total prediction error: $\mathrm{err}(n) = \|y - H\hat{\theta}_{\mathrm{LS}}^{(n)}\|^2$, where y is the actual speed vector and $\hat{\theta}_{\mathrm{LS}}^{(n)}$ is the least squared estimate of the parameters. Plot err vs. n. Based on your plot, choose a reasonable n and justify your choice. For the chosen n, plot your predicted wind speed and the actual wind speed on the same figure. (10pts)

2. **E911:** The federal government has mandated that cellular network operators must have the ability to locate a cell phone from which an emergency call is made. This problem concerns a simplified version of an E-911 system that uses time of arrival information at a number of base stations to estimate the cell phone location. A cell phone at location $x \in \mathbb{R}^2$ (we assume that the elevation is zero for simplicity) transmits an emergency signal at time. This signal is received at n base stations, located at locations $s^{(1)}, s^{(2)}, \ldots, s^{(n)} \in \mathbb{R}^2$. Each base station can measure the time of arrival of the

emergency signal, within a few tens of nanoseconds. This is possible because the base stations are synchronized using the Global Positioning System. The measured times of arrival are

$$t_i = \frac{1}{c}\|s^{(i)} - x\| + \tau + v_i, i = 1, \dots, n, \tag{2}$$

where $c = 0.3$m/ns is the speed of light, and v_i is the noise or error in the measured time of arrival. The problem is to estimate the cell phone position $x \in \mathbb{R}^2$, as well as the time of transmission τ, based on the time of arrival measurements $t_1, \dots, t_n$. Positions of the 9 base stations $s^{(i)} = (x_i, y_i)$ and the measured times of arrival t_i are given in the Table 1. Distances are given in meters, times in nanoseconds, and the speed of light in meters/nanosecond. You can assume that the position x is somewhere in the box $|x_1| \leq 3000$, $|x_2| \leq 3000$, and that $|\tau| \leq 5000$.

i	1	2	3	4	5	6	7	8	9
x_i (m)	-20000	1000	-3000	4000	-3500	1000	4000	-4000	6000
y_i (m)	5000	3500	-4000	1000	2000	6000	-3000	-1500	-1000
t_i (ns)	79445	20009	21622	13683	24709	28223	11293	22990	16446

Table 1: Data for the E911 problem.

(a) Let $\theta = [x_1, x_2, \tau]^T$. Formulate the E911 problem as a nonlinear least square problem. (3pts)

(b) Solve the nonlinear least square problem. Please attach your codes and provide the numerical values of the obtained estimate. (4pts)

3. **Robot Dynamics Simulation and Parameter Identification:** Consider a planar 2R open chain moving in the presence of gravity. The chain moves in the $\hat{x}$-$\hat{y}$-plane, with gravity g acting in the $-\hat{y}$-direction. To keep things simple, the two links are modeled as point masses m_1 and m_2 concentrated at the ends of each link. We choose the joint coordinates $\theta = [\theta_1, \theta_2]^{\mathrm{T}}$ as the generalized coordinates. The generalized forces $\tau = [\tau_1, \tau_2]^{\mathrm{T}}$ then correspond to joint torques. Then then dynamic equations for the 2R planar chain are given by

$$\begin{aligned}
\tau_1 = & \left(m_1 L_1^2 + m_2\left(L_1^2 + 2L_1L_2\cos\theta_2 + L_2^2\right)\right)\ddot{\theta}_1 + m_2\left(L_1L_2\cos\theta_2 + L_2^2\right)\ddot{\theta}_2 \\
& - m_2L_1L_2\sin\theta_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right) + (m_1+m_2)L_1 g\cos\theta_1 + m_2 g L_2\cos(\theta_1+\theta_2) \\
\tau_2 = & m_2\left(L_1L_2\cos\theta_2 + L_2^2\right)\ddot{\theta}_1 + m_2L_2^2\ddot{\theta}_2 + m_2L_1L_2\dot{\theta}_1^2\sin\theta_2 + m_2 g L_2\cos(\theta_1+\theta_2).
\end{aligned} \tag{3}$$

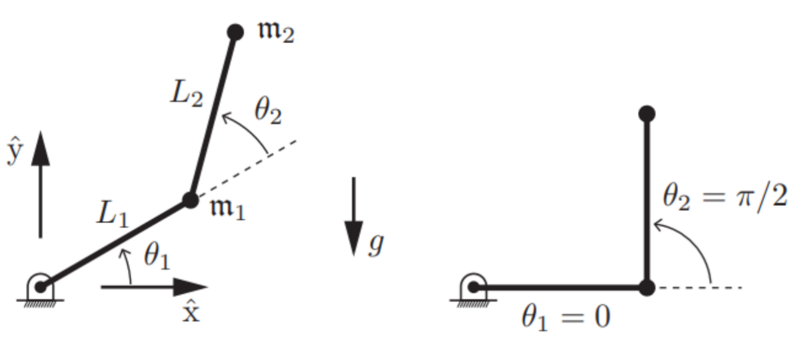

Figure 1: (Left) A 2R open chain under gravity. (Right) At $\theta = \left[0, \frac{\pi}{2}\right]^{\mathrm{T}}$.

We can gather terms together into an equation of the form

$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta), \tag{4}$$

with

$$M(\theta) = \begin{bmatrix} m_1 L_1^2 + m_2\left(L_1^2 + 2L_1L_2\cos\theta_2 + L_2^2\right) & m_2\left(L_1L_2\cos\theta_2 + L_2^2\right) \\ m_2\left(L_1L_2\cos\theta_2 + L_2^2\right) & m_2L_2^2 \end{bmatrix}$$

$$c(\theta, \dot{\theta}) = \begin{bmatrix} -m_2L_1L_2\sin\theta_2\left(2\dot{\theta}_1\dot{\theta}_2 + \dot{\theta}_2^2\right) \\ m_2L_1L_2\dot{\theta}_1^2\sin\theta_2 \end{bmatrix}$$

$$g(\theta) = \begin{bmatrix} (m_1 + m_2)L_1 g\cos\theta_1 + m_2 g L_2\cos(\theta_1 + \theta_2) \\ m_2 g L_2\cos(\theta_1 + \theta_2) \end{bmatrix}$$

where $M(\theta)$ is the symmetric positive-definite mass matrix, $c(\theta, \dot{\theta})$ is the vector containing the Coriolis and centripetal torques, and $g(\theta)$ is the vector containing the gravitational torques. These reveal that the equations of motion are linear in $\ddot{\theta}$, quadratic in $\dot{\theta}$, and trigonometric in θ.

(a) Based on the Eq.(4), derive the continuous-time state space model for the 2R robot dynamics, and then obtain the discrete time state space model with $T = 0.001$. (4pts)

(b) Write a function that can simulate the "free falling" (i.e. zero joint torque) of the 2R robot (Hint: the parameters of the function are the initial states of the robot). Test your function with 2 different initial states (i. $\theta_1 = 0, \theta_2 = \frac{\pi}{2}$; ii. $\theta_1 = 0, \theta_2 = -\frac{\pi}{2}$). Visualize your simulation by drawing a 2R robot (Assuming $m_1 = m_2 = 1$, $L_1 = L_2 = 0.5$, $g = 9.81$). (4pts)

(c) Now suppose you do not know the system parameters (m_1, m_2, L_1, L_2), and you can measure $\tau, \theta, \dot{\theta}, \ddot{\theta}$ at sampling times. Write a program to implement a least squared estimation of the dynamic parameters. Test your program using your simulator in part (b) with perfect measurement. Discuss your results. (10pts)