

# ME424 Project #2

Due Nov 21, 2022

Assigned Nov 13, 2022

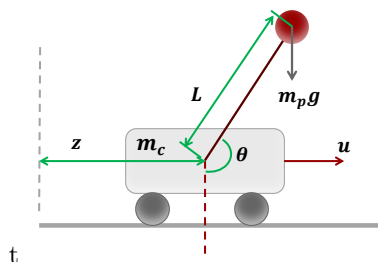
- Please attach all of your codes and relevant figures. To receive credits, please write down all the necessary steps leading to your final solution. Make sure your discussions about the results are clear, brief, and to the point.
- Please form a team with 2 or 3 people. Each team needs to turn in one report. Please make sure everyone in the team is involved and understand the solutions. You need to attach the contribution of each member (in percentage) in your report.
- This course has a zero-tolerance policy on plagiarism.

Consider a cart of mass  $M$  slides on a horizontal frictionless track, and is pulled by a horizontal force  $u(t)$ . On the cart an inverted pendulum of mass  $m_p$  is attached via a frictionless hinge, as shown in the figure below. The pole's center of mass is located at a distance  $L$  from the revolute joint located at the center of mass of the cart. The variable  $\theta(t)$  is the angle that the pendulum makes with respect to the vertical axis. The dynamics of the system is given by:

$$\begin{cases} \ddot{z} = \frac{1}{m_c + m_p \sin^2 \theta} \left[ u + m_p \sin \theta \left( L \dot{\theta}^2 + g \cos \theta \right) \right] \\ \ddot{\theta} = \frac{1}{L(m_c + m_p \sin^2(\theta))} \left[ -u \cos(\theta) - m_p L \dot{\theta}^2 \cos \theta \sin \theta - (m_c + m_p) g \sin \theta \right] \end{cases} \quad (1)$$

where  $m_p = 1$ ,  $m_c = 10$ ,  $g = 9.81$ , and  $L = 0.5$ .

1. **Full state space model:** Define the state vector:  $x = [z, \theta, \dot{z}, \dot{\theta}]^T$ . Derive the continuous time state space model  $\dot{x} = f(x, u)$  based on the dynamic relation given in equation (1).
2. **Angle dynamics model:** In this project, we will design control to stabilize the angle  $\theta$ . For controller design, we only need to capture the angle dynamics. Now let  $x_\theta = [\theta, \dot{\theta}]^T$  and write down the nonlinear dynamics for  $\dot{x}_\theta = f_\theta(x_\theta, u)$  based on the complete model developed Part 1.
3. **Linearized model for angle dynamics:** Consider the angle dynamics model obtained in Part 2. Derive the linearized continuous time model around  $(\hat{x}_\theta, \hat{u}) = ([0, 0]^T, 0)$ , i.e., find  $\hat{A}$ ,  $\hat{B}$ , such that  $\dot{x}_\theta = f_\theta(x_\theta, u) \approx \hat{A}x_\theta + \hat{B}u$ . (note that  $f_\theta(\hat{x}_\theta, \hat{u}) = 0$ ,  $\Delta x_\theta \triangleq x_\theta - \hat{x}_\theta = x_\theta$ ,  $\Delta u \triangleq u - \hat{u} = u$ ).
4. **Discrete Time Angel Dynamics Model:** Based on the continuous-time linearized model in Part 3, find the matrices  $A, B, C$  for the discrete time model  $x_\theta(k+1) = Ax_\theta(k) + Bu(k)$ ,  $y(k) = Cx_\theta(k)$ , under the sampling time  $T = 0.005$ . Assume that  $y(k) = \theta(k)$ , i.e., you can only measure the angle.
5. **Drake Simulation Setup and Testing**
  - (a) Follow Tutorial 4 to install Drake
  - (b) Download the "Proj2.ipynb" file from github. Simulate the uncontrolled cart-pole system with at least two sets of initial conditions. Attach your codes and a few snapshots of your simulation.



**6. Simulation studies for state-feedback control**

- (a) **(Eigenvalue assignment):** Use the model  $(A, B, C)$  derived in part 4 to design a linear feedback controller  $u = -Kx_\theta$ . You should compute  $K$  such that  $A - BK$  has eigenvalue  $z_{1,2} = e^{s_{1,2} * T}$ , where  $s_{1,2} = -2 \pm j$ .
  - (b) **(Closed-loop Simulation):** Implement your controller in Drake (as a child class of LeafSystem). Note the control  $u = -Kx_\theta$  is based on the reduced-order state  $x_\theta$  instead of the complete state  $x$ . Simulate & visualize the response for a few different initial conditions. Attach your codes and a few snapshots/plots.
7. Repeat the design and the corresponding closed-loop simulation with a new set of desired continuous time poles  $s_{1,2} = -2 \pm 8j$ . Compare the responses with the previous case and discuss your observations.
8. **Output feedback control design:** Now suppose we do not measure all the state  $x_\theta$ . Our output measurement is only the angle:  $y = \theta$
- (a) Suppose your desired observer eigenvalues in continuous time are:  $s_{1,2} = -9 \pm 2j$ , find the corresponding discrete-time eigenvalues and find the observer gain  $L$  so that  $A - LC$  has the desired discrete time observer eigenvalue.
  - (b) Simulate the entire output-feedback control system in Drake and visualize the response for a few different initial conditions. For controller, please use the control gain  $K$  computed in Part 5. For the Luenberger observer implementation, you can either use the built-in LinearSystem class and develop from scratch as a new child class of LeafSystem.