ME424 Fall 2022 | **Project # 3**
Assigned December 17, 2022 | Due: December 26, 2022

- Please work on this project by **yourself independently**.
- Please attach all of your codes and relevant figures. To receive credits, please write down all the necessary steps leading to your final solution. Make sure your discussions about the results are clear, brief, and to the point.
- You must type your report. Overall presentation and organization count.

Kalman Filter: The dynamic equation of a series RLC circuit (see Figure 1) can be written as

$$\begin{aligned} u + w &= IR + L\dot{I} + V_c \\ I &= C\dot{V}_c \end{aligned} \tag{1}$$

where u is the applied voltage signal and w is the noise of the source voltage with $w(t) \sim \mathcal{N}(0, 0.25)$ for all $t \in \mathbb{R}$. Let $R = 3$, $L = 1$ and $C = 0.5$.

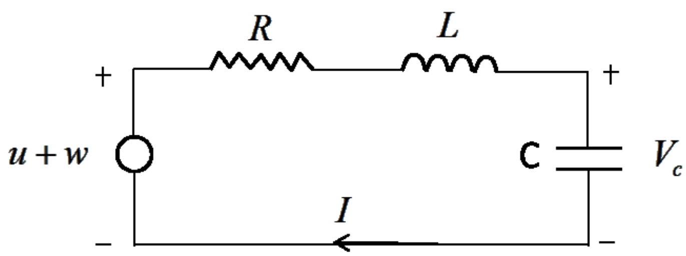

Figure 1: Series RLC circuit

1. Let $x = [I,\ V_c]^T$. Find the continuous-time state space model of the form $\dot{x} = A_c x + B_c u + G_c w$ by computing the matrices A_c, B_c and G_c.

2. With a discretization step dt, one can approximate the continuous time model as $x(t + dt) = x(t) + (A_c x(t) + B_c u(t) + G_c w(t))dt$. Using this strategy, let $u_k = u(k \cdot dt)$, $x_k = x(k \cdot dt)$, obtain a discrete time model of the form: $x_{k+1} = A_d x_k + B_d u_k + \tilde{w}_k$ under a 10-Hz sampling speed ($dt = 0.1$s). You need to specify A_d, B_d and $\mathrm{Cov}(\tilde{w}_k)$.

3. Suppose at each discrete time k, the capacitor voltage V_c is measured with Gaussian white noise with unitary variance. This corresponds to $y_k = [0,\ 1]x_k + v_k$, with $\mathrm{Cov}(v_k) = 1$. Let $u_k = \cos(4\pi k/200)$ and simulate the system for 200 steps (with $x_0 = [0,\ 0]^T$) and save your x_k and y_k as ground truth.

4. Implement the Kalman filter (with $\hat{x}_0 = [0, 0]^T$ and $P_0 = 0.1I$) in Python for this RLC system using the data generated in part (c). Plot the simulated and estimated currents in the same figure; plot the simulated and estimated V_c in another figure. Plot the measurement error v_k and estimation error for V_c, i.e. $\hat{x}_{k,2} - x_{k,2}$, on the same plot. Brief explain your results.

5. Now suppose you don't know the input signal u, and you simply assume the state update model is $x_{k+1} = Ax_k + z_k$ where $z_k \sim \mathcal{N}(0, \sigma I)$. Modify your Kalman filter according to this new state update model and compute the state estimates using the same data generated in (c) for $\sigma = 0.001, 0.01, 0.02, 0.1$. Plot the estimated voltage $\hat{x}_{k,2}$ for these 4 cases on top of the true voltage $x_{k,2}$. Please also compute the average squared error $\frac{1}{200}\sum_{k=1}^{200}(x_{k,2} - \hat{x}_{k,2})^2$ for all these four cases. Briefly explain your observations.