MEE5114 Advanced Control for Robotics

Lecture 5: Instantaneous Velocity of Moving Frames

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Outline

- Instantaneous Velocity of Rotating Frames

- Instantaneous Velocity of Moving Frames

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Instantaneous Velocity of Rotating Frame (1/2)

- {A} frame is rotating with orientation $R_A(t)$ and velocity $\omega_A(t)$ at time t (Note: everything is wrt {O}-frame)

- Let $\hat{\omega}\theta = \log(R_A(t))$ be its exp. coordinate.
  - Note: $\hat{\omega}\theta$ means $R_A(t)$ can be obtained from the reference frame (say {O}-frame) by rotating about $\hat{\omega}$ by θ degree.
  - $\hat{\omega}\theta$ only describes the current orientation of {A} relative to {O}, it does not contain info about how the frame is rotating at time t.

Instantaneous Velocity of Rotating Frame (2/2)

- What is the relation between $\omega_A(t)$ and $R_A(t)$?

$$\frac{d}{dt}R_A(t) = [\omega_A(t)]R_A(t) \Rightarrow [\omega_A(t)] = \dot{R}_A(t)R_A^{-1}(t)$$

Outline

- Instantaneous Velocity of Rotating Frames
- Instantaneous Velocity of Moving Frames

Instantaneous Velocity of Moving Frame (1/2)

- {A} moving frame with configuration $T_A(t)$ at time t undergoes a rigid body motion with velocity $\mathcal{V}_A(t) = (\omega, v)$ (Note: everything is wrt {O}-frame)

- The exponential coordinate $\hat{\mathcal{S}}\theta = \log(T_A(t))$ only indicates the current configuration of {A}, and does not tell us about how the frame is moving at time t.

Instantaneous Velocity of Moving Frame (2/2)

- What is the relation between $\mathcal{V}_A(t)$ and $T_A(t)$?

$$\frac{d}{dt}T_A(t) = [\mathcal{V}_A(t)]T_A(t) \Rightarrow [\mathcal{V}_A(t)] = \dot{T}_A(t)T_A^{-1}(t)$$

More Space