MEE5114 Spring 2023 **Homework # 5**

Assigned April 13, 2023 Due: April 19, 2023

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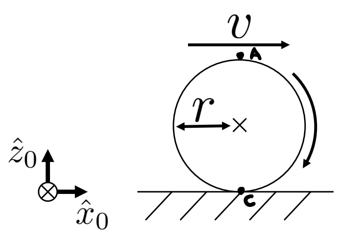
1. A cylinder rolls without slipping in the $\hat{x}_0$ direction. The cylinder has a radius of r and a constant forward speed of v. What is the spatial acceleration of this cylinder expressed in {o}, ${}^o\mathcal{A}$ and expressed in {C}, ${}^C\mathcal{A}$, where frame {C} has the same orientation as frame {o} and its origin is at the contact point C.



2. Let ${}^OT_{A=(R,p)}$ be the pose of frame A. Suppose A is moving with velocity ${}^O\mathcal{V}_{A=(\omega,v)}$. Show that
$$\frac{d}{dt}\left[{}^OX_A^*\right] = \begin{bmatrix} [\omega] & [v] \\ 0 & [\omega] \end{bmatrix} {}^OX_A^*$$

3. A rigid body is a collection of point masses m_i as location p_i. Given a reference point o, the angular momentum of point mass i is $\overrightarrow{op_i} \times m_i v_i$. Given the definition of the angular momentum of the rigid body $\phi_o = \sum_i \overrightarrow{op_i} \times m_i v_i$, show that for any reference point o and q, we have
$$\phi_q = \phi_o + \overrightarrow{qo} \times L$$
where L is the linear momentum of the rigid body.

4. Given our derivation in class, we have $M(\theta) = \sum_i J_i^T \mathcal{I}_i J_i$ and $c(\theta, \dot{\theta}) = \sum_i J_i^T (\mathcal{I}_i \dot{J}_i + \mathcal{I}_i \mathcal{V}_i \times J_i + \mathcal{V}_i \times^* \mathcal{I}_i J_i)$. Prove that $\dot{M} - 2c$ is skew symmetric.